

Air Traffic Flow Management in the Presence of Uncertainty

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Abstract— Deterministic air traffic flow management (TFM) decisions—the state of the art in terms of implementation—often result in “lost” airspace capacity, because the inherent uncertainties in weather predictions make it difficult to determine the number of aircraft that can be safely accommodated in a volume of airspace during a given period. As a result, there is a distinct need for TFM algorithms that utilize available stochastic weather information for improved decision making. To this end, we first develop a methodology to determine the stochastic capacity for a volume of airspace given the forecast weather and associated uncertainty. Then, we use this information as input to a dynamic stochastic optimization algorithm to determine the number of aircraft to send towards a volume, providing specific guidance for routing aircraft in the presence of the uncertainties of adverse weather.

Keywords—air traffic flow management; stochastic capacity; dynamic routing; weather

I. INTRODUCTION

En route convective weather is a stochastic phenomenon which, when severe, prevents aircraft from operating in parts of the National Airspace System (NAS). As a result, aircraft that are already airborne must be either held in the air in holding patterns or re-routed around adverse weather events, and aircraft waiting to depart must either be held on the ground or re-routed once they are in the air. Because weather conditions evolve over time, re-routing and holding decisions are based on predictions of the location, timing, and severity of adverse weather events.

Due to the significant uncertainty inherent in weather predictions, current traffic flow management (TFM) methods often result in lower utilization levels of airspace capacity. For

example, airspace capacity is lost when aircraft are re-routed to longer paths, which may later become blocked, while shorter paths through the weather system, that materialize after the re-routing decisions are made, go unused. Similarly, airspace capacity is also lost when aircraft are held on the ground for longer than actually necessary while they could have traveled through the adverse weather on paths that did become available. In truth, some of this lost capacity will be recovered as the accuracy of weather predictions improve. However, it is not likely, given the history of weather prediction accuracy, that weather prediction uncertainty will be eliminated or reduced to the point where deterministic TFM will result in full utilization of available capacity. Thus, there is a distinct need for TFM algorithms that can account for weather uncertainties.

Stochastic approaches to TFM mostly exist in the context of the Ground Holding Problem (GHP). However, GHP only accounts for airport arrival/departure capacities, ignoring en route capacity constraints. In addition, most GHP models have focused on the static single-airport GHP. There are only a few studies that address dynamic stochastic versions of the TFM problem. Reference [1] proposes a multistage stochastic integer programming formulation with recourse options for GHP. Reference [2] proposes a stochastic dynamic approach with en route capacities where they model the problem of routing an individual flight across weather impacted regions as a Markov decision process. Reference [3] extends this model to multiple aircraft. In another such study, [4] suggests a multistage stochastic programming formulation based on a set of capacity scenarios for a single airport.

These proposed stochastic approaches have several limitations, most important of which is the lack of a mapping between weather forecasts and capacity. While most models assume some given capacity measure without studying the impact of weather conditions, [2] tries to account for the dynamics of weather directly in their problem formulation. However, this leads to the oversimplification of weather

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dynamics, as well as additional computational complexity in the algorithm. Some other limitations in the existing studies include the intractability of the proposed formulations when realistically large scenario sets are considered, and the assumption that only a fixed set of alternative routes are available for rerouting purposes, which may again lead to lower utilization of airspace capacity.

In this study, we fill these significant gaps in air traffic flow management by developing a comprehensive and robust dynamic optimization procedure that accounts for the uncertainty in weather conditions and its impact on airspace capacity, while at the same time maintaining practical applicability and computational tractability. To this end, we first develop algorithms that translate available weather forecast information to stochastic airspace capacity, and then provide specific guidance for routing aircraft based on this calculated capacity. With this guidance, increased traffic levels can be handled under different weather scenarios compared to today's way of traffic management which does not deal with probabilistic information on weather.

Our approach to stochastic TFM has two main components: (1) conversion of weather forecasts into capacity information, i.e. traffic flow targets, for a volume of airspace such as a sector; and (2) usage of stochastic capacity information to dynamically route aircraft destined to that airspace. The first component includes a weather scenario generation algorithm, along with a traffic generator and a fuel-optimal conflict resolution algorithm. The second component is based on a dynamic implementation of a stochastic programming model, which is solved efficiently to allow for application in real-time.

In the remainder of this paper, we describe each component of the proposed TFM methodology in detail. In Section II, we present the algorithms that we have developed to convert probabilistic weather forecast data into stochastic capacity information. More specifically, we describe the weather scenario generation tool, the conflict resolution algorithm, and the simulations performed using these tools. In Section III, we present the stochastic routing algorithm and its implementation. Finally, the conclusions are outlined in Section IV, along with possible extensions to the paper.

II. DETERMINING THE STOCHASTIC CAPACITY OF A VOLUME OF AIRSPACE

If the effects of weather on airspace capacity are to be mitigated through effective TFM, we must first characterize the relationship between different weather patterns and capacity, where we define capacity as the maximum number of aircraft that can enter a volume of airspace and transit without having to be diverted to another volume of airspace or to an airport during a fixed time interval. This, however, is a complicated problem due to the large number of aircraft involved, the number of decision makers, and more importantly the uncertainty of weather information.

Despite these complications, there have been some efforts to model the relationship between weather and airspace capacity. Reference [5] performs a clustering analysis on historical weather data and identifies seven categories of

weather patterns. They then select a representative day for each category, and study the impact of weather conditions on that day. To judge the capacities of the weather-impacted sectors, the authors use a route-based approach, in which a deterministic sector capacity is defined to be proportional to the fraction of routes that were unavailable. This estimate is then adjusted for uncertainty by adding a randomized bias term to serve as the mean capacity level at a future time. The capacity is assumed to be normally distributed, and the standard deviation is modeled as a function of the look-ahead time and the weather pattern. Reference [6] suggests an alternate approach. In their study, the authors first define the sector capacity as a function of the traffic flow pattern, and then attempt to capture the impact of weather on sector capacity by calculating route blockage probabilities through probabilistic weather forecasts. In the sector-specific study by [7], separate models are developed for ten highly congested Air Traffic Control (ATC) sectors. Several past weather events are identified as being particularly problematic and used as the scenarios in the study. Route blockage probability calculations are performed using the algorithm developed by [8], based on observed weather parameters and traffic patterns within these sectors. A probability density function for sector route blockage was then derived using conditional probabilities described by [9].

One shortcoming of these weather-capacity models is that they either simplify the complexities that need to be considered or they focus on specific ATC sectors. The reality is that weather patterns vary significantly throughout the NAS. Thus, any model of the relationship between weather and capacity must be able to capture all these differences if it is to be broadly applicable. Further, none of the proposed approaches make use of the probabilistic convective weather forecasts that are available. We attempt to remedy these shortcomings by developing a generic model that can be configured to reflect any of the observed relationships between convective weather and capacity for a given volume of airspace.

The weather-capacity model is developed in three steps. In the first step, an algorithm is devised to generate scenarios representing the evolution of convective weather in spatial and temporal dimensions, based on probabilistic weather forecasts. In the second step, we develop a fuel-optimal conflict resolution algorithm, designed to solve for conflict-free trajectories within an airspace. In the third and final step, Monte Carlo simulations of the traffic flow are conducted over the range of possible weather and traffic scenarios using the conflict-resolution algorithm developed. For a given traffic arrival distribution and arrival rate, each simulation run either yields a conflict-free flow within the airspace over a predefined time interval or results in an infeasible case, where conflict-free flow is not possible for that scenario. By performing multiple runs at increasing levels of arrival rate and analyzing the ratio of the cases with feasible versus infeasible flow, we derive the probability distribution of capacity for the airspace over that time interval. These three steps of the weather-capacity model are further explained below.

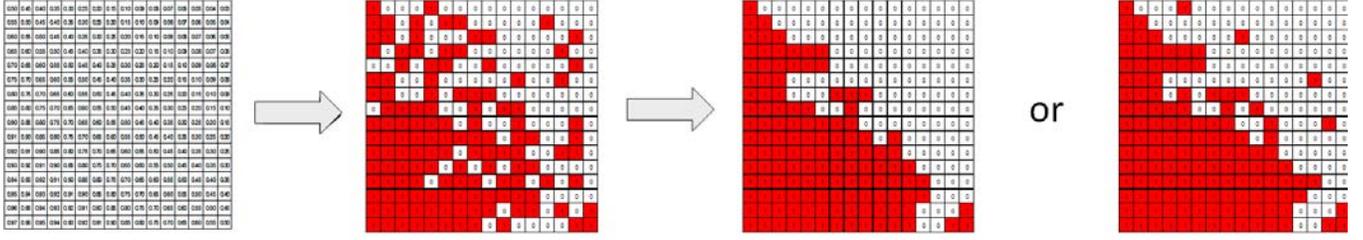


Figure 1. Mapping from a probability matrix to a cell blockage map: simple random sampling followed by basic smoothing or adaptive smoothing.

A. Weather Scenario Generation using Probabilistic Weather Forecasts

In this section, we develop an easy-to-implement weather model that we use within the context of the simulation analysis described above, to study the evolution of a given weather event over multiple time intervals in an airspace. More specifically, the model is used to generate scenarios of weather movements. The only inputs to the model are the probabilistic weather forecasts, such as the 1-6 hour National Convective Weather Forecast (NCWF-6) [10, 11]. The probabilistic forecasts of convection are based on blending radar-based extrapolation forecasts and Rapid Update Cycle (RUC)-based Convective Probability Forecasts (RCPF) of convection [12]. The forecast probabilities indicate the likelihood that aviation disrupting convection (e.g., radar echo exceeding 35 dBZ) will be present at a given location at the given forecast time [10]. The forecast probabilities are provided for certain time intervals (e.g. 15 or 30 minutes) 6 hours into the future and are updated every 15 minutes. The nominal spatial resolution is the same as the latest RUC, i.e. 13 km.

The modeling process involves a mapping from forecast probabilities valid at a sequence of forecast time intervals to traffic blockage maps at a sequence of work time intervals, where blockage means that the area is not available for air traffic. Each simulated blockage map sequence is an instantiation of the stochastic process. It is required that as the number of simulated sequences becomes large, the number of instances a cell is blocked at a given time divided by the total number of instance sequences simulated should approach the forecast probability for that cell, at that time interval. Based on this requirement, the simulated ensemble of traffic blockage map sequences would be representative of what would occur, as the number of simulated sequences becomes large.

Once a sequence of probability matrices on the work grid and at work time intervals is obtained, the mapping from these continuous probability matrices to binary blockage maps can be performed. This is achieved by passing a band limited 2D random signal (such as a uniformly distributed random signal) defined on the work grid through a shaping filter determined by the sequence of probability matrices. For each of the probability matrices, a corresponding binary cell blockage map is generated as shown in Fig. 1. Conflict resolution and re-routing algorithms can then be applied based on the sequence of cell blockage maps corresponding to the sequence of probability matrices. However, if simple random sampling is

used, such generated blockage maps will lack the spatial correlation between cells (i.e. clustering of blocked cells) and the temporal evolution between successive cell blockage maps (i.e. continuous trending, growth, and decay of the weather system). Hence, to model the spatial correlation between cells, we apply smoothing techniques to the 2D random signal, before it is modulated by the probability matrix to generate the blockage map. Then, we use cellular automata techniques to model the transition of blockage maps from one time interval to the next. We now provide the details of the smoothing and the cellular automata techniques utilized in this process.

1) *Modeling Spatial Correlation:* To model the spatial correlation between cells, Gaussian smoothing is applied to the 2D random matrix generated to map the probabilistic weather information to a cell blockage map. The strength of the spatial correlation between cells can be presented by the Full Width at Half Maximum (FWHM) of the Gaussian kernel. FWHM is related to the standard deviation of the Gaussian kernel by the following equation:

$$\text{FWHM} = \sigma\sqrt{8\ln 2}. \quad (1)$$

In the third matrix from left in Fig. 1, it can be seen that the blocked cells are more clustered after basic smoothing, which is more likely to be representative of the actual conditions, when a weather system is present. On the other hand, using a fixed FWHM Gaussian kernel across the whole grid may prevent scattered cell blockage in low probability areas. To enable scattered (or popup) cell blockage, we use adaptive smoothing. In this approach, the FWHM of the Gaussian kernel used for each individual cell is dynamically adapted to reflect the varying strength of spatial correlation between cells. First, a temporal cell blockage map is obtained via direct mapping without smoothing. The percentage q_{ij} of blocked neighboring cells (including the cell itself) is then calculated for each cell, based on this cell blockage map. The FWHM_{ij} for a cell (i, j) is then calculated as a function of q_{ij} . The size of the neighborhood used in calculating q_{ij} , and the functional relationship between the percentage of blocked neighboring cells and the FWHM are determined using historical data. In general, for a cell with a higher percent of blocked neighboring cells, a larger FWHM is used to indicate stronger spatial correlation between cells. For a cell with a lower percent of blocked neighboring cells, a smaller FWHM

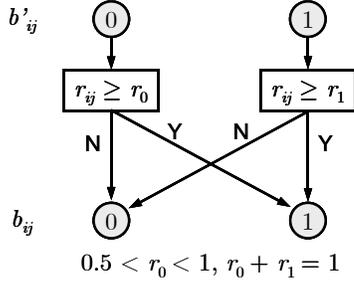


Figure 2. Cellular automata used to adjust cell blockage

is used to indicate weaker spatial correlation between cells. This would allow retaining scattered blockage as can be seen in the fourth matrix from left in Fig. 1, which is not possible by using a single FWHM across the whole grid. Alternatively, the percentage q_{ij} of blocked neighboring cells can be calculated based on the cell blockage map at the previous time interval.

As in the case of the spatial correlation between cells at any given time period, the transition between cell blockage maps at consecutive time periods must also be modeled, for which we use the cellular automata methods described below.

2) *Modeling Temporal Correlation:* The cellular automaton that we develop does not directly determine the states of cells, i.e. cell blockage maps, but rather, it is used to modify the cell blockage maps obtained using the mapping process presented in the previous section. For each cell, the percentage r_{ij} of blocked neighboring cells at the previous time interval is calculated first. If the cell blockage map obtained using the adaptive smoothing mapping process is $[b'_{ij}]$, then state b_{ij} of each cell at the current time interval is determined by the process shown in Fig. 2.

The transition process shown in the figure implies that the cell state from the mapping process will be accepted as is, if it is in the same state as the majority of its neighboring cells at the previous time interval. The threshold r_0 for converting an unblocked cell from the mapping process to a blocked cell must be greater than 0.5. The threshold r_1 for converting a blocked cell from the mapping process to an unblocked cell must be less than 0.5. The summation of the two thresholds is required to be 1 to ensure the probability of cell blockage is preserved. The transition rules can also be presented in the following equation.

$$b_{ij} = [r_{ij} \geq (1 - b'_{ij})r_0 + b'_{ij}r_1]. \quad (2)$$

Application of the process described above will result in the transition of cell states from one time interval to the next occurring near the boundary of convection. Additional special rules are also developed to allow for the popup and growth of scattered blocked cells in low probability areas. Size of neighborhoods and transition thresholds are again determined using historical data.

In Fig. 3, we summarize the steps used to generate scenarios that model the spatial and temporal evolution of convective weather, based on probabilistic weather forecasts.

B. Fuel-Optimal En Route Conflict Resolution Algorithm

An efficient conflict resolution algorithm is required to determine the maximum number of aircraft that can safely transit through a volume of airspace within a predefined time interval for each weather scenario that is generated. Although conflict resolution is a well-studied topic in the literature [13], none of the existing algorithms are suitable for implementation in a simulation environment, mainly due to computational issues. Such an algorithm has to be very fast and efficient, as multiple large instances of the problem need to be solved over several replications. Despite the suitability of rule-based methods for such fast calculations, an optimization based approach is necessary to ensure that simulations result in accurate capacity distributions. However, the corresponding optimization problem is complex, and there are no available comprehensive models with efficient solution methods for the problem.

Hence, we develop a novel conflict resolution algorithm which is capable of resolving conflicts through fast numerical optimization methods, through simultaneous heading and speed changes. In addition, particular focus has been placed on reducing fuel costs involved in conflict resolution. This was deemed to be important, given the significant role that fuel plays in the operating cost of aircraft and the growing concern regarding the impact of gaseous emissions on the environment. In addition to implementations in simulation environments, due to its efficiency, the proposed model can also be implemented to resolve conflicts in near real-time.

As part of the description of the model, we first provide a general problem statement and a mathematical formulation, and then present the solution methodology, along with some sample results.

1) *Problem Statement:* Consider a set of n aircraft located in a Euclidean plane. Each aircraft i is defined by an initial position $\mathbf{p}_i = (x_i, y_i)$, a velocity vector $\mathbf{v}_i^0 = (v_{i,x}^0 + v_{i,y}^0)$ defining speed and heading, and a desired final heading θ_i^d . Additionally, all aircraft are designated to be a particular model type with corresponding fuel burn characteristics for the given altitude. The objective of the conflict resolution problem is to assign each aircraft a single instantaneous heading and speed change at $t=0$, such that the aircraft will travel conflict-free, while minimizing a measure of the fuel burn costs over all the trajectories. Note that the algorithm can be implemented iteratively to ensure conflict free transit through a volume of airspace, assuming that the aircraft would maneuver back to their initial destinations after clearance of conflicts.

The trajectory of an aircraft i is deemed to be conflict free, if the distance between aircraft i and any other aircraft j , $d_{ij} = d_{ji}$, is always greater than the minimum distance of d_{ij}^{min} . The minimum separation distance can be visualized by encircling each aircraft with a safety region of radius $d/2$. If

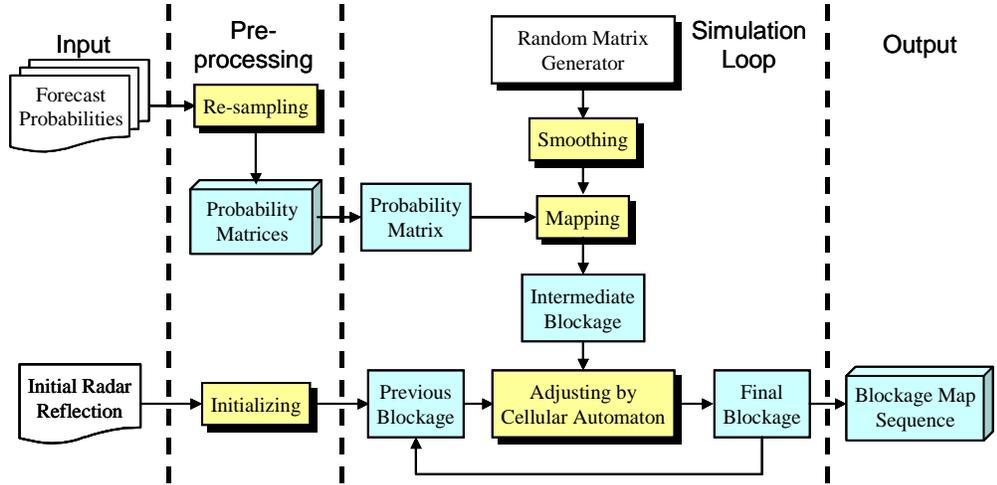


Figure 3. Procedure used to generate weather scenarios based on probabilistic weather forecasts.

we assume that trajectories of aircraft are linearly extrapolated in time, then for aircraft i and j with given trajectories, the necessary minimum separation condition is expressed by the following inequality:

$$\sqrt{x_{dist}^2 + y_{dist}^2} \geq d_{ij}^{\min} \quad \forall t \in \mathbb{R}^+ \quad (3)$$

where x_{dist} and y_{dist} represent the distance between the two aircraft in the corresponding coordinate axes, and are defined by:

$$\begin{aligned} x_{dist} &= (x_i + v_{i,x}t) - (x_j + v_{j,x}t) \\ y_{dist} &= (y_i + v_{i,y}t) - (y_j + v_{j,y}t) \end{aligned} \quad (4)$$

We now describe a methodology for formulating a fuel-optimal conflict resolution model that ensures that separation conditions (3) hold at all times. Unlike most models in the literature, the process yields a mixed integer linear programming problem, which is solvable in near real time for dynamic routing decisions.

Starting with the initial conditions $\{(\mathbf{p}_i, \mathbf{v}_i^0)\}$, the solution to the resulting optimization model will be the set of new velocity vectors $\{\mathbf{v}_i^+\}$ for each aircraft. Speed and heading commands ensuring that separation conditions hold, can then be extracted from \mathbf{v}_i^+ . The model does not take into account the time to execute heading and speed changes, as it is assumed that deviations are small and the time to complete any maneuver fix is small in comparison to time to the conflict. However, the safety region about each aircraft can be expanded to handle uncertainty from resulting maneuver changes, wind variation, or other unmodeled phenomena.

2) *Model Formulation:* We first present the mathematical representation of the constraints that need to be enforced to ensure a conflict-free solution, and show that they can be expressed as a set of linear inequalities. Then we develop very

tight approximations for the nonconvex cost functions that represent fuel consumption.

a) *Separation constraints:* Consider a pair of aircraft i and j with initial position and velocity states:

$$\begin{aligned} \mathbf{p}_i &= (x_i, y_i), \quad \mathbf{v}_i^0 = [v_{i,x}^0, v_{i,y}^0]^T \\ \mathbf{p}_j &= (x_j, y_j), \quad \mathbf{v}_j^0 = [v_{j,x}^0, v_{j,y}^0]^T \end{aligned} \quad (5)$$

A given aircraft i may alter its trajectory to prevent conflict by changing its velocity vector by $\mathbf{d}\mathbf{v}_i = [dv_{i,x}, dv_{i,y}]^T$. Applying $\mathbf{d}\mathbf{v}_i$ to each corresponding aircraft defines new trajectories as follows:

$$\mathbf{v}_i^+ = \mathbf{v}_i^0 + \mathbf{d}\mathbf{v}_i, \quad \mathbf{v}_j^+ = \mathbf{v}_j^0 + \mathbf{d}\mathbf{v}_j \quad (6)$$

The set of linear constraints ensuring that a pair of aircraft maintains separation is derived from the relative velocity $\tilde{\mathbf{v}}_{i,j}$ and initial position $\tilde{\mathbf{p}}_{i,j}$ of aircraft i to aircraft j , i.e.

$$\begin{aligned} \tilde{\mathbf{v}}_{i,j} &= \mathbf{v}_i^{+,r} - \mathbf{v}_j^{+,r} \\ \tilde{\mathbf{p}}_{i,j} &= \mathbf{p}_i^{+,r} - \mathbf{p}_j^{+,r} \end{aligned} \quad (7)$$

Conflicts between aircraft i and aircraft j occur when the ray originating from aircraft i extending along $\tilde{\mathbf{v}}_{i,j}$ passes through aircraft j . To ensure separation, an implementation based on the definition of a safety region around each aircraft is possible. For aircraft with safety regions of radius $d/2$, the projected safety region of aircraft i along $\tilde{\mathbf{v}}_{i,j}$ must remain outside the safety region of aircraft j , as illustrated in Fig. 4. By understanding the method of ray extension along the relative velocity, the allowable regions for $\tilde{\mathbf{v}}_{i,j}$ can be delineated. Ultimately, a set of crossing lines, $l_{i,j}^p$ and $l_{i,j}^n$, with slopes $m_{i,j}^p$ and $m_{i,j}^n$, tangent to the safety regions of each

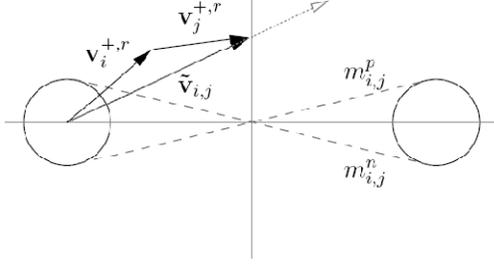


Figure 4. Definition of the safety regions for aircraft.

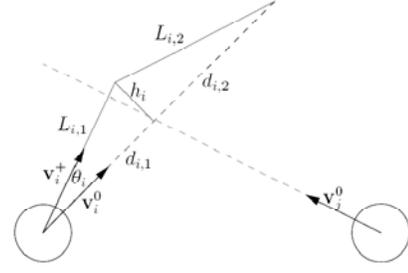


Figure 5. Heading angle deviations increase the resulting distance traveled

aircraft are key to defining the linear constraints through the following relation:

$$\frac{\tilde{v}_{i,j,y}}{\tilde{v}_{i,j,x}} \leq m_{i,j}^n \quad \text{or} \quad \frac{\tilde{v}_{i,j,y}}{\tilde{v}_{i,j,x}} \geq m_{i,j}^p \quad (8)$$

Constraints (8) can be expressed as linear inequalities by multiplying the right-hand side by the denominator. Keeping mindful of the condition $\tilde{v}_{i,j,x} = 0$, separation is then ensured when:

$$\begin{aligned} \tilde{v}_{i,j,y} &\leq \tilde{v}_{i,j,x} m_{i,j}^n, & \tilde{v}_{i,j,x} &\geq 0 & \text{or} \\ \tilde{v}_{i,j,y} &\geq \tilde{v}_{i,j,x} m_{i,j}^p, & \tilde{v}_{i,j,x} &\geq 0 & \text{or} \\ \tilde{v}_{i,j,x} &\leq 0 & & & \end{aligned} \quad (9)$$

The separation constraints (9) can easily be expressed as linear inequalities of the decisions variables $\tilde{v}_{i,j,x}$ and $\tilde{v}_{i,j,y}$ which are functions of the speed and heading changes, d_{vi} . The constraints are then applied to all pairs of aircraft. As the constraints are reciprocal, only one set of constraints is required for each pair. To model the avoidance required to route an aircraft around a no-fly region, such as convective weather, we use the same process to form a set of linear constraints.

b) Cost functions: In order to provide a framework in which fuel costs are considered in conflict resolution and aircraft routing, we define a fuel-burn cost function $G_0(\mathbf{s}, \theta)$ as:

$$G_0(\mathbf{s}, \theta) = g_s(\mathbf{s}) + g_h(\theta) \quad (10)$$

where g_s and g_h are nonlinear scalar functions of the airspeeds \mathbf{s} , and the headings θ of the aircraft. The function g_s measures the fuel burn percentage of an aircraft, while g_h accounts for the scaled increase in distance traveled due to a deviation from the desired route. Considering both parts, G_0 is the fuel consumption percentage with respect to the optimal path at a desired airspeed when there are no obstacles.

The measures \mathbf{s} and θ , and thus the cost functions $g_s(\mathbf{s})$ and $g_h(\theta)$ are nonlinear nonconvex functions of the decision variables $d\mathbf{v}_i$. However, we develop tight convex linear approximations for these functions, and show that the underlying optimization problem can be modeled as a linear integer programming problem.

To model the fuel costs due to the change in airspeed, i.e. $g_s(\mathbf{s})$, we consider an aircraft with some initial heading θ^0 , and which can perform heading changes of $\mp d\theta$ to ensure separation, consistent with typical air traffic management procedures. We assume that the range of possible final heading values is broken into m adjacent regions according to the set $\theta = \{d\theta_1 + \theta^0, \dots, \theta^0, \dots, d\theta_m + \theta^0\} = \{\theta_1, \dots, \theta_m\}$. These regions need not be uniform in size. A grid structure over the feasible space is then formed including the origin, and the set $(X_q, Y_q) = \{v_i^{max} \cos(\theta_q), v_i^{max} \sin(\theta_q)\}$, $\forall q \in \{1, 2, \dots, m\}$. The function $Z_q = \|\mathbf{v}_q\|$ is then evaluated over the grid points. The airspeed, \hat{s}_i , is calculated by forming a convex combination of the function values of the grid points associated with the sector encompassing \mathbf{v}_i^+ , and is given by the following set of constraints:

$$\begin{aligned} \hat{v}_{i,x}^+ &= \sum_{q=0}^m X_q \lambda_q \\ \hat{v}_{i,y}^+ &= \sum_{q=0}^m Y_q \lambda_q \\ \hat{s}_i &= \sum_{q=0}^m Z_q \lambda_q \\ \sum_{q=0}^m \lambda_q &= 1 \\ \lambda_q &\in \text{SOS2} \end{aligned}$$

where SOS2 is the specially ordered set of type 2. The SOS2 approximation yields a very tight approximation over the domain. Computational tests show that even with only four regions, spread over ± 45 degrees around the initial heading, the percent error between the approximation and the actual airspeed is always less than 2%.

The fuel cost associated with a heading angle deviation and a return to the desired flight path is approximated using a two step process, as shown in Fig. 5. In the first step, the aircraft makes a heading angle change to resolve conflict, and then in the second stage the heading is corrected back towards the destination. Let $Di = d_{i,1} + d_{i,2}$ designate the straight-line distance between the destination and the current position of the airplane i . If maintaining separation requires a heading

angle change, then the travel distance is $L_{i,1} + L_{i,2}$, and the scaled increase $D_{p,i}$ in the travel distance is $D_{p,i} = (L_{i,1} + L_{i,2}) / D_i$. Substituting in terms of the heading change $d\theta$ for $L_{i,1}$ and applying the law of cosines to solve for $L_{i,2}$ for $d\theta_i \in (-\pi/2, \pi/2)$, $D_{p,i}$ can be represented as:

$$D_{p,i}(d\theta_i) = \frac{\frac{d_{i,1}}{\cos(d\theta_i)} + \sqrt{\left(\frac{d_{i,1}}{\cos(d\theta_i)}\right)^2 + D_i^2 - 2d_{i,1}D_i}}{D_i} \quad (11)$$

Assuming that any heading angle change allows for the aircraft to fly near the optimal fuel burn speed, the additional distance traveled according to relation (11) can be represented by a fuel consumption measure. Note that equation (11) is a convex function of $d\theta_i$ in the interval of interest. Thus, a linear approximation is possible by fitting a set of $2q$ planes between angles $[\theta_{-q}, \dots, \theta^0, \dots, \theta_q]$ to (11). Each plane w , approximating the function (11) within some interval $[\theta_w, \theta_{w+1}]$ can be determined by first calculating the points x_w, y_w, z_w and $x_{w+1}, y_{w+1}, z_{w+1}$ as follows:

$$\begin{aligned} x_w &= v_{i,max} \cos(\theta_w) \\ y_w &= v_{i,max} \sin(\theta_w) \\ z_w &= D_{p,i}(\theta_w) \\ x_{w+1} &= v_{i,max} \cos(\theta_{w+1}) \\ y_{w+1} &= v_{i,max} \sin(\theta_{w+1}) \\ z_{w+1} &= D_{p,i}(\theta_{w+1}) \end{aligned} \quad (12)$$

Then, a linear function relating the scaled increase in distance traveled due to a heading deviation can be obtained by:

$$\det \begin{pmatrix} x & y & \hat{D}_{p,i}^w - 1 \\ x - x_w & y - y_w & \hat{D}_{p,i}^w - z_w \\ x - x_{w+1} & y - y_{w+1} & \hat{D}_{p,i}^w - z_{w+1} \end{pmatrix} = 0 \quad (13)$$

where $\hat{D}_{p,i}^w$ is the approximate percent increase in distance traveled and $x = v_{i,x}^+$ and $y = v_{i,y}^+$. The resulting relation can be included as a constraint in the optimization model as $\hat{D}_{p,i}^w = c_1x + c_2y + c_3$, where c_1 , c_2 and c_3 are constants obtained through (13). Computational test show that the approximation error for this convex planar representation of the nonlinear objective function is less than 1% for almost all values of the heading angle change within the nominal operating bounds.

We assume that the total cost is a mixture of costs to minimize the sum of individual fuel costs and minimize the maximum fuel-burn over all the aircraft (to ensure that no single aircraft is excessively penalized). Thus, the overall objective function is expressed as a weighted sum of these two objectives.

Due to the novel approximation techniques described above, the resulting model can be implemented as a mixed integer programming problem. A detailed discussion of this

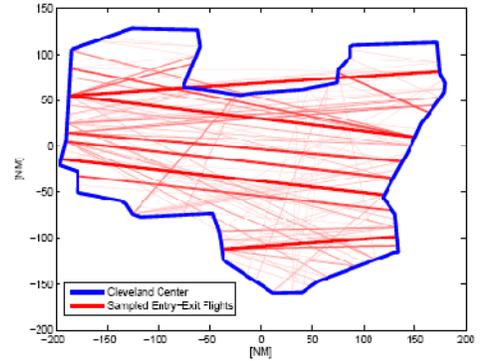


Figure 6. Randomly generated flights in Cleveland Center based on distributions calculated from historical data for simulation purposes

model and proofs for the validity of the derived relationships for the algorithm are described in [14]. One significant aspect of the developed algorithm is that the optimal conflict resolution maneuvers are identified in only a few seconds, which enables the implementation of this methodology as part of the simulations required for capacity distribution calculations.

C. Derivation of Stochastic Capacity through Monte Carlo Simulation

The next step in determining the capacity distribution for a volume of airspace under potential convective weather is to simulate traffic flow in this airspace based on the weather scenarios generated and a given traffic pattern.

The implementation of the simulation for a given volume of airspace can be performed as follows. To model the traffic, a statistical distribution of entry-exit pairs is generated using historical data of aircraft flying through the airspace. The boundary of the airspace is then broken into segments, which are numerically identified. For the distribution, each aircraft is designated to enter and exit through a particular boundary segment. Although any distribution can be used to model aircraft interarrival times into the airspace, we assume that they follow an exponential distribution, with a slight modification such that aircraft entering at the same entrance must have a minimum time separation, based on current separation requirements. An example of the sampled traffic pattern through Cleveland center is displayed in Fig. 6. In order to account for different fuel burn rates, aircraft models, e.g. regional, narrow body, wide body, and business class jets, can also be assigned according to a sampled distribution taken from the historical data.

The simulations are conducted such that for a given expected arrival rate, arriving traffic into the volume of airspace based on the assumed distribution, and the convective weather based on the sampled scenarios is simulated over a fixed time period. Throughout the simulation, the conflict resolution algorithm is utilized to dynamically route aircraft around convective weather in a fuel-optimal and conflict-free manner. However, based on the random evolution of traffic and weather, some instances are likely to lead to congestion,

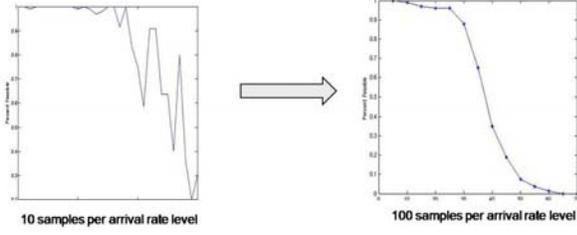


Figure 7. Increased number of replications result in convergence of simulation results to form an empirical cdf of capacity

for which conflict-free trajectories are not possible without diverting one or more aircraft to a different volume of airspace or an airport. These instances are identified by the conflict resolution algorithm as infeasible.

We perform multiple replications of the simulation at different expected arrival rate values, and record the ratio of feasible solutions provided by the algorithm over the total number of replications for each expected arrival rate value. As the arrival rate is increased, this ratio gradually approaches zero, resulting in an empirical cumulative probability distribution for given probabilistic weather forecast information. The convergence of the simulation results to a smooth empirical distribution is demonstrated in Fig. 7.

This cumulative distribution can then be converted into a probability density or mass function of airspace capacity for a given time interval, which can directly be used in a stochastic

$$\begin{array}{l}
 P(\text{cap} \leq 10) = 0.01 \\
 P(10 < \text{cap} \leq 20) = 0.03 \\
 P(20 < \text{cap} \leq 30) = 0.07 \\
 P(30 < \text{cap} \leq 40) = 0.54 \\
 P(40 < \text{cap} \leq 50) = 0.25 \\
 P(50 < \text{cap} \leq 60) = 0.08 \\
 P(60 < \text{cap}) = 0.02
 \end{array}
 \begin{array}{l}
 \rightarrow P(\text{low cap})=0.04 \\
 \rightarrow P(\text{medium cap})=0.61 \\
 \rightarrow P(\text{high cap})=0.35
 \end{array}$$

TFM algorithm. In most cases, a discrete distribution may be preferred for better practical interpretation of the information on the stochastic parameter. As an example, we can calculate the probabilities of predefined low, medium and high capacity levels according to the derived distribution in Fig. 7 as follows:

Derivation and availability of such capacity distribution information has utmost importance for air traffic flow management, as it will lead to more robust planning with significantly improved utilization of available capacity.

III. STOCHASTIC PROGRAMMING BASED DYNAMIC ROUTING ALGORITHM FOR TFM

The next step in the developed stochastic TFM procedures is the determination of the number of aircraft to send towards a volume of airspace given the capacity distributions. For this purpose, we develop a stochastic programming model, which considers flights on the ground and in the air, destined to the considered volume of airspace. The model determines the departure time with cruise speed and ground delay decisions for these flights as well as any holding and diversion decisions

that may be necessary after the realization of convective weather.

Costs corresponding to each of these actions are introduced and calculated in the model for each flight. In addition, we assume limits on the allowable number of time periods for each flight to be ground-delayed or air-held, reflecting practical constraints such as fuel availability and crew schedules. Similarly, changes in airspeed are also limited to some extent depending on the aircraft specifications. Further, we make some additional assumptions on the problem. First, we model the problem by using discrete time intervals, where a 15-minute interval may be used for typical implementation. Similarly, we assume that discrete probability distributions of airspace capacity are available, resulting in the definition of a set of capacity scenarios over the planning period.

The following notation is used in the description of the stochastic programming model and the proposed solution procedure:

- M : the set of flights
- S : the set of time periods
- b_m : scheduled departure period for flight $m \in M$
- Δk_m : scheduled flight time of flight m from current position to the sector
- Δs_m : maximum number of periods that flight m can be ground-delayed
- Δt_m^+ : maximum number of periods that flight m can be scheduled to arrive early
- Δt_m^- : maximum number of periods that flight m can be scheduled to arrive late
- Δh_m : maximum number of periods that flight m can be held in the air
- g_m^s : ground delay cost for flight m , if it departs at time s
- $c_m^{t-s-\Delta k_m}$: speed change cost for flight m , if its arrival time at the sector is changed from $s + \Delta k_m$ to t
- a_m^{u-t} : air holding cost for flight m , if it arrives the sector at time t and enters at time u
- d_m : cost of diverting flight m
- C^u : capacity of the sector at time u
- x_m^{st} : 1, if flight m departs at time s and arrives at the sector at time t ; 0, otherwise
- q_m^t : 1, if flight m arrives at the sector and is diverted at time t ; 0, otherwise
- p_m^{tu} : 1, if flight m arrives at the sector at time t and enters at time u ; 0, otherwise

Using this notation, we develop the following stochastic integer program:

$$\begin{aligned}
\min \quad & \sum_{m \in M} \left\{ \sum_{s=b_m}^{b_m+\Delta s_m} \sum_{t=s+\Delta k_m-\Delta t_m^+}^{s+\Delta k_m+\Delta t_m^-} \left[(g_m^s + c_m^{t-s-\Delta k_m}) \cdot x_m^{st} \right] \right. \\
& \left. + \sum_{t=b_m+\Delta k_m-\Delta t_m^+}^{b_m+\Delta s_m+\Delta k_m+\Delta t_m^-} \left[d_m \cdot \mathbf{E}[\mathbf{Q}_m^t(\xi)] + \sum_{u=t}^{t+\Delta h_m} (a_m^{u-t} \cdot \mathbf{E}[\mathbf{P}_m^{tu}(\xi)]) \right] \right\} \\
\text{s.t.} \quad & \sum_{m \in M} \sum_t p_m^{tu}(\xi) \leq C^u \quad \forall u \\
& q_m^t(\xi) + \sum_{u=t}^{t+\Delta h_m} p_m^{tu}(\xi) = \sum_s x_m^{st} \quad \forall t, m \\
& \sum_{s=b_m}^{b_m+\Delta s_m} \sum_{t=s+\Delta k_m-\Delta t_m^+}^{s+\Delta k_m+\Delta t_m^-} x_m^{st} = 1 \quad \forall m \\
& x_m^{st} \in \{0, 1\} \quad \forall s, t, m \\
& p_m^{tu}(\xi), q_m^t(\xi) \in \{0, 1\} \quad \forall t, u, m
\end{aligned}$$

The objective function of the above model is the minimization of the expected total cost, which is represented as a sum of the first stage ground-delay and speed-change costs, and the expected air-holding and diversion costs in the second stage. The expectation is defined as the sum of these costs over all scenarios, weighted by the scenario probabilities. The first set of constraints ensures that sector capacity is not exceeded in each time period. The second set of constraints in the model requires that if a flight is sent towards the sector, it must either be diverted or allowed into the sector. The third constraints enforce that a schedule is determined for each flight. Without loss of generality, we do not model the cancellation of a flight before departure. However, that and other similar decisions can easily be incorporated in the model.

For a large number of aircraft and scenarios, the stochastic model above becomes inapplicable for real-time implementation. Hence, it is necessary that an efficient procedure is utilized to enable fast calculations for dynamic routing of aircraft. To this end, we develop a rolling horizon implementation, which significantly reduces the computational time and provides a near-optimal solution.

The rolling horizon method solves the described stochastic programming problem in a sequential order, by dividing it into several smaller problems. To ensure consistency in the routing decisions, the sector capacity is updated using the previous iteration's solution at each iteration. More specifically, in the first iteration a set of flights is selected and solved to optimality. Based on these solutions, sector capacity is updated for the second iteration. Then, another set of flights, which may also include flights from the previous iteration, e.g. those that were held on the ground throughout prior periods in which decisions were made, is selected and solved to optimality. The procedure is repeated until all flights are considered and their schedules are fixed. The update on the capacity constraints can be performed by replacing the second constraint of the above model with

$$\sum_{m \in M} \sum_t p_m^{tu}(\xi) \leq C^u - \sum_{m \in M} \sum_{t'} p_m^{t'u}(\xi) \quad \forall u$$

where the right hand side is updated according to the flights whose schedules are fixed in previous iterations.

TABLE I. PERFORMANCE OF THE ROLLING HORIZON METHOD

flights depart in period	num. of flight	Solve to Optimality			Rolling Horizon Method			
		objective value	Time (sec)	period	modified objective value	Time (sec)	number of iteration	% from optimal
1-2	27	2,644.95	11.12	5-13	2,644.95	4.00	1	0.00%
1-3	28	2,647.95	371.96	5-16	2,647.95	4.00	1	0.00%
1-4	31	3,403.17	422.59	5-16	3,478.06	5.00	1	2.20%
1-5	35	3,417.01	1,247.56	5-18	3,564.01	5.00	2	4.30%
1-6	41	n/a	n/a	5-19	5,287.55	5.00	3	n/a
1-7	48	n/a	n/a	5-19	9,367.37	11.00	4	n/a

Dependent on the flights selected and their flight times, only a limited number of periods may be considered in each iteration of the algorithm, which further speeds up the implementation. In addition, different rules can be applied when selecting the set of flights to freeze their schedules or to reconsider in the iterations. This set of rules can be determined based on factors such as the required accuracy levels in the approximations and the available computational tools.

The devised solution method is very efficient in solving large instances of the stochastic TFM problem with minimal levels of optimality gap, when compared to standard solution methods, i.e. to solving the deterministic equivalent of the problem as a whole. This can be seen in the computational results presented in Table 1 for a sample problem with 19 periods and up to 48 flights, where a maximum of 2^{14} capacity scenarios were considered.

Overall, the developed procedure is a fast and dynamic stochastic optimization method that uses as input the stochastic capacity information calculated for a volume of airspace in determining the number of aircraft to send towards that airspace. Its implementation will clearly lead to TFM procedures that perform better than the currently used deterministic decisions.

IV. CONCLUSIONS

In this paper, we developed a comprehensive air traffic flow management methodology which will increase airspace capacity and utilization by providing a mathematically rigorous basis for determining optimal or near-optimal solutions to the problem of traffic flow management in the presence of uncertainty.

Specifically, we first devised an algorithm to calculate probabilistic predictions of capacity in en route sectors based on probabilistic weather forecasts. Then, we developed a stochastic programming based procedure to determine how much to reduce the spatial and temporal buffers that currently decrease capacity in en route airspace. The overall study demonstrates the value of the availability of probabilistic weather forecasts, as well as the value of the utilization of a stochastic decision making tool for air traffic flow management.

Extensions to the study include the generalization of the results by simultaneous considerations of multiple volumes of

airspace and implementation of a multi-stage decision process in determining optimal routings. Further, the capacity estimations can be parameterized so that it is possible to map a given probabilistic weather matrix to a capacity distribution. This will enable even faster decision making, and will allow more time for human intervention in a fully automated system.

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